Bernoulli’s Infinity Argument

Let $R$ be the number of ways of getting a success in a single trial and let $S$ be the number of ways of getting a failure in a single trial, and let $T = R + S$. Let $W_i$ be the number of ways of getting exactly $i$ successes in $NT$ trials.

When Bernoulli calculates $\frac{W_{NR}}{W_{NR+N}}$ which in his notation is $M/L$, he gets

$$M/L = \frac{NRS + NS}{NRS - NR + R} \times \frac{NRS + NS - S}{NRS - NR + 2R} \times \ldots \times \frac{NRS + S}{NRS}$$

Dividing the numerators and denominators by $N$, he gets

$$M/L = \frac{RS + S}{RS - R + \frac{R}{N}} \times \frac{RS + S - \frac{S}{N}}{RS - R + \frac{2R}{N}} \times \ldots \times \frac{RS + \frac{S}{N}}{RS}$$

He says as $N$ approaches infinity we can ignore all the terms at the end of each numerator and denominator which are divided by $N$.

So he gets

$$\frac{RS + S}{RS - R} \times \frac{RS + S}{RS - R} \times \ldots \times \frac{RS}{RS}$$

So since $\frac{RS + S}{RS - R} > 1$, then as $N$ approaches infinity,

$$[(RS + S)/(RS - R)]^N$$ also approaches infinity.

He concludes that $M/L$ approaches infinity as $N$ approaches infinity.

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